AEROSPACE REPORT NO.
ATR-88(3728-02)-1

High-Temperature Creep Under a Nonuniform Temperature Distribution

Prepared by

L. A. FELDMAN
Materials Sciences Laboratory
The Aerospace Corporation

and

T. B. BAHDER
U.S. Army Laboratory Command
Harry Diamond Laboratory
Adelphi, MD 20783-1197

15 May 1989

Laboratory Operations
THE AEROSPACE CORPORATION
El Segundo, CA 90245

Prepared for

OFFICE OF NAVAL RESEARCH
800 North Quincy Street
Arlington, VA 22217

5

19960628 127

THE AEROSPACE CORPORATION

APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED

LABORATORY OPERATIONS

The Aerospace Corporation functions as an "architect-engineer" for national security projects, specializing in advanced military space systems. Providing research support, the corporation's Laboratory Operations conducts experimental and theoretical investigations that focus on the application of scientific and technical advances to such systems. Vital to the success of these investigations is the technical staff's wide-ranging expertise and its ability to stay current with new developments. This expertise is enhanced by a research program aimed at dealing with the many problems associated with rapidly evolving space systems. Contributing their capabilities to the research effort are these individual laboratories:

Aerophysics Laboratory: Launch vehicle and reentry fluid mechanics, heat transfer and flight dynamics; chemical and electric propulsion, propellant chemistry, chemical dynamics, environmental chemistry, trace detection; spacecraft structural mechanics, contamination, thermal and structural control; high temperature thermomechanics, gas kinetics and radiation; cw and pulsed chemical and excimer laser development including chemical kinetics, spectroscopy, optical resonators, beam control, atmospheric propagation, laser effects and countermeasures.

Chemistry and Physics Laboratory: Atmospheric chemical reactions, atmospheric optics, light scattering, state-specific chemical reactions and radiative signatures of missile plumes, sensor out-of-field-of-view rejection, applied laser spectroscopy, laser chemistry, laser optoelectronics, solar cell physics, battery electrochemistry, space vacuum and radiation effects on materials, lubrication and surface phenomena, thermionic emission, photosensitive materials and detectors, atomic frequency standards, and environmental chemistry.

Computer Science Laboratory: Program verification, program translation, performance-sensitive system design, distributed architectures for spaceborne computers, fault-tolerant computer systems, artificial intelligence, microelectronics applications, communication protocols, and computer security.

Electronics Research Laboratory: Microelectronics, solid-state device physics, compound semiconductors, radiation hardening; electro-optics, quantum electronics, solid-state lasers, optical propagation and communications; microwave semiconductor devices, microwave/millimeter wave measurements, diagnostics and radiometry, microwave/millimeter wave thermionic devices; atomic time and frequency standards; antennas, rf systems, electromagnetic propagation phenomena, space communication systems.

Materials Sciences Laboratory: Development of new materials: metals, alloys, ceramics, polymers and their composites, and new forms of carbon; nondestructive evaluation, component failure analysis and reliability; fracture mechanics and stress corrosion; analysis and evaluation of materials at cryogenic and elevated temperatures as well as in space and enemy-induced environments.

Space Sciences Laboratory: Magnetospheric, auroral and cosmic ray physics, wave-particle interactions, magnetospheric plasma waves; atmospheric and ionospheric physics, density and composition of the upper atmosphere, remote sensing using atmospheric radiation; solar physics, infrared astronomy, infrared signature analysis; effects of solar activity, magnetic storms and nuclear explosions on the earth's atmosphere, ionosphere and magnetosphere; effects of electromagnetic and particulate radiations on space systems; space instrumentation.

Aerospace Report No.
ATR-88(3728-02)-1

HIGH-TEMPERATURE CREEP UNDER A NONUNIFORM TEMPERATURE DISTRIBUTION

Prepared by

L. A. Feldman
Materials Sciences Laboratory
The Aerospace Corporation

and

T. B. Bahder
U.S. Army Laboratory Command
Harry Diamond Laboratory
Adelphi, MD 20783-1197

15 May 1989

Laboratory Operations
THE AEROSPACE CORPORATION
El Segundo, CA 90245

Prepared for

OFFICE OF NAVAL RESEARCH
800 North Quincy Street
Arlington, VA 22217

Report No.
ATR-88(3728-02)-1

HIGH-TEMPERATURE CREEP UNDER A
NONUNIFORM TEMPERATURE DISTRIBUTION

Prepared

L. A. Feldman

L. A. Feldman

L. A. Feldman for

T. B. Bahder

Approved

H. Katzman

H. A. Katzman, Head
Carbon and Polymers Department

R. W. Fillers

R. W. Fillers, Director
Materials Sciences Laboratory

ACKNOWLEDGMENTS

We thank Dr. Paul Blatz for helpful discussions.

Funding for this effort was processed through SD Contract No. F04701-85-C-0086-P00019 under an Interagency Agreement from the Office of Naval Research.

The dimensional stability of a material often plays an important role in the use of that material at high temperature, particularly in structural applications. The determination of creep behavior at high temperature is frequently complicated by a nonuniform temperature distribution. In order to approximate the ideal situation of a uniform temperature distribution, various schemes are used, such as reducing the cross section within the hot zone to restrict the highest stress to a suitably small region. However, in many practical situations, as well as in experiments dealing with the measurement of creep in composites (Ref. 1), a material is inevitably subjected to a nonuniform temperature distribution. The total creep elongation is then a sum of contributions from material elements with a distribution of temperatures.

In this report, we analyze an example of creep in a material under a nonuniform temperature distribution, and we use simple but realistic assumptions for that distribution and for the material creep properties as a function of temperature. This analysis was motivated by measurements of creep at high temperatures in unidirectional, fibrous carbon-carbon composites (Refs. 2 and 3).

A sample of uniform cross section was used because of ease of fabrication. The sample was placed in a furnace with a central peak temperature. A tensile load was applied to the sample and creep elongation was measured. However, creep rates versus temperature were not obtainable directly from the creep data, due to the temperature distribution across the sample. The following analysis allowed us to predict both creep rates at a uniform temperature and elongation rates under other, nonuniform temperature distributions.

The sample in our model is long and thin (depicted schematically in Fig. 1), having length 2L and uniform cross-sectional area A in the high-temperature region. We assume that all strains are small, $\varepsilon \ll 1$, and that the sample is under tension, P, or uniaxial tensile stress, $\sigma = P/A$. Creep rate, $\dot{\varepsilon}(x)$, and the temperature distribution, $T(x)$, are functions of position along the sample axis, x.

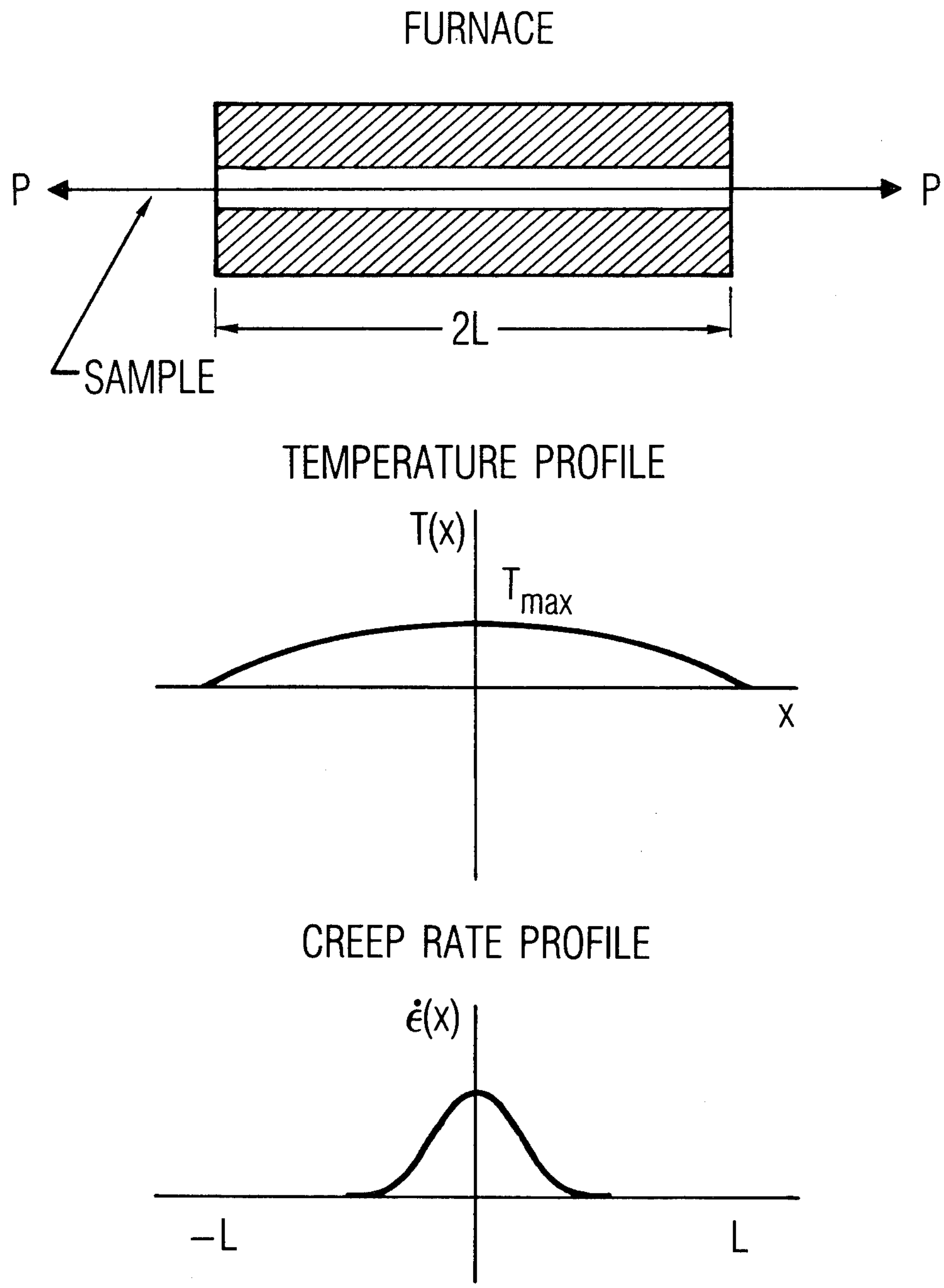

Fig. 1. Schematic of idealized experiment.

The average creep rate over the sample length is given by

$$\langle \dot{\varepsilon} \rangle = \frac{1}{2L} \int_{-L}^{L} \dot{\varepsilon}(x)\, dx \tag{1}$$

We assume the material has a simple Dorn-Weertman (Ref. 4) temperature-dependent creep behavior:

$$\dot{\varepsilon} = \dot{\varepsilon}_0(\sigma)\, \exp(-E/kT) \tag{2}$$

where $\dot{\varepsilon}_0(\sigma)$ is a rate coefficient, σ is stress, E is activation energy, k is Boltzmann's constant, and T is absolute temperature. A simple expression for the average creep rate under a nonuniform temperature distribution is obtained by inserting Eq. (2) into Eq. (1):

$$\langle \dot{\varepsilon} \rangle = \frac{1}{2L} \int_{-L}^{L} \dot{\varepsilon}_0(\sigma)\, \exp[-E/kT(x)]\, dx \tag{3}$$

We also assume the parabolic temperature distribution (Refs. 3 and 5):

$$T(x) = T_{max}\left[1 - \left(\frac{x}{L}\right)^2\right] \tag{4}$$

with a peak temperature, T_{max}. For convenience, the temperature at the ends of the sample is taken to be absolute zero, which is a reasonable approximation because the contribution to the integral from the regions of the sample near the ends is small. We define $\alpha = E/kT_{max}$ to be a dimensionless activation energy, and eliminate x in favor of q, a dimensionless variable, in Eqs. (3) and (4) through the substitution $x/L = [1 + (\alpha/q)]^{-1/2}$. Using the fact that T(x) is an even function of x, we then obtain

$$\frac{\langle \dot{\varepsilon} \rangle}{\dot{\varepsilon}_0(\sigma)} = \frac{1}{2}\, \alpha^{-1/2}\, e^{-\alpha} \int_0^{\infty} e^{-q}\, q^{-1/2} \left(1 + \frac{q}{\alpha}\right)^{-3/2} dq \tag{5}$$

This integral can be easily evaluated by noting the results of experiments on high-temperature creep in carbon-carbon composites (Ref. 2). Activation energies of the order of 100 kcal/mole were obtained at temperatures of around 2500 K, thus giving values of α of ~20. Consequently, we can expand the binomial in the integrand in powers of q/α and integrate term by term. Because the integral is over all $q > 0$, we integrate the series outside its radius of convergence, which results in an expression for the integral that is an asymptotic (rather than convergent) series (Ref. 6) in powers of $1/\alpha$:

$$\frac{\langle\dot{\varepsilon}\rangle}{\dot{\varepsilon}_0(\sigma)} = \frac{1}{2}\left(\frac{\pi}{\alpha}\right)^{1/2} e^{-\alpha} \sum_{k=0}^{\infty} (-1)^k \left(\frac{1}{\alpha}\right)^k \frac{(2k+1)!\,(2k)!}{(k!)^3\, 2^{4k}}$$

$$= \frac{1}{2}\left(\frac{\pi}{\alpha}\right)^{1/2} e^{-\alpha}\left[1 - \frac{3}{4}\left(\frac{1}{\alpha}\right) + \frac{45}{32}\left(\frac{1}{\alpha}\right)^2 - \frac{525}{128}\left(\frac{1}{\alpha}\right)^3 + \ldots\right] \qquad (6)$$

An alternative approach to evaluating the integral in Eq. (5) is the method of steepest descents (Ref. 6), an approach that gives the first term in the series in Eq. (6).

For experimentally relevant values of α, the series in Eq. (6) gives a good approximation to the integral, to within a few percent, with only the first two terms.* For a value of α ~20, it can be seen from Eq. (6) that the first-order term in $1/\alpha$ gives a correction of less than 4%. This suggests in fact that, to a reasonable approximation,

$$\frac{\langle\dot{\varepsilon}\rangle}{\dot{\varepsilon}_0(\sigma)} = \frac{1}{2}\left(\frac{\pi}{\alpha}\right)^{1/2} e^{-\alpha} \qquad (7)$$

Using Eq. (7) to analyze the elongation rates at different peak temperatures, we can obtain the activation energy, E, and pre-exponential $\dot{\varepsilon}_0(\sigma)$.

*In general, best precision is obtained with the series by locating the minimum term and truncating the series one term before it, see Ref. 6.

The result in Eq. (7) can be viewed in terms of an "effective gauge length," L_{eff}, defined by

$$\langle\dot{\varepsilon}\rangle L = \dot{\varepsilon}_0(\sigma)e^{-\alpha}L_{eff} = \dot{\varepsilon}L_{eff} \tag{8}$$

where $\dot{\varepsilon}$ is the strain rate for a sample under a uniform temperature equal to T_{max}, given in Eq. (2).

For the temperature distribution in Eq. (4),

$$L_{eff} = \frac{1}{2}\left(\frac{\pi}{\alpha}\right)^{1/2}L \tag{9}$$

Thus, in a typical experimental example (Refs. 2 and 3), for $\alpha = 20$ and a sample length, 2L, of 91.4 cm (36 in.),

$$L_{eff} = \frac{1}{2}\left(\frac{\pi}{20}\right)^{1/2}(45.7 \text{ cm}) = 8.9 \text{ cm } (3.5 \text{ in.})$$

Equation (2) can be used with L_{eff} to obtain $\dot{\varepsilon}(2L_{eff})$, which is the elongation rate, and which gives total elongation when multiplied by the time interval.

A problem mathematically similar to that of calculating creep under a nonuniform temperature distribution is determining the amount of diffusion or degree of heat treatment when the sample is held at a uniform temperature, but the temperature varies as a function of time, T(t). For example, following Shewmon (Ref. 7), the mean-squared diffusion distance is expressed as

$$\langle x^2\rangle = 2\,D(T)t \tag{10}$$

where the diffusion constant D(T) is typically of the form

$$D = D_0\,\exp(-E/kT)$$

If temperature varies with time as $T(t)$, then the total mean-squared diffusion distance over a time t' is

$$\langle x^2 \rangle = 2 \int_0^{t'} D[T(t)]\, dt \tag{11}$$

In conclusion, for thermally activated creep we have shown that an asymptotic expansion for the average elongation rate can be used to determine the creep parameters $\alpha = E/kT_{max}$ and the pre-exponential factor $\dot{\varepsilon}_0(\sigma)$. We have done this for a parabolic temperature distribution, but the same approach can be used for other temperature distributions. Having determined the parameters, we can use the expression in Eq. (3) to predict the elongation rate for any given temperature distribution. In addition, we have indicated that analogous problems, such as heat treatment, are of similar mathematical form.

REFERENCES

1. S. Mack and G. Sines, High Temperature Mechanical Testing of a Cylindrical Weave Carbon-Carbon Composite, Report UCLA-ENG-85-20, UCLA School of Engineering, Los Angeles, Calif. (July 1, 1985).

2. L. A. Feldman, in Extended Abstracts, 17th Biennial Conference on Carbon (University of Kentucky, Lexington, 16-21 June 1985), pp. 393-394.

3. L. A. Feldman, High Temperature Creep of Carbon Yarns, Report TOR-0084A(5728-02)-1, The Aerospace Corporation, El Segundo, Calif. (15 July 1985).

4. H. W. Hayden, W. G. Moffatt, and J. Wulff, Mechanical Behavior, Vol. 3 in Structure and Properties of Materials, Wiley, New York (1965), p. 134.

5. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, 2nd ed., Oxford University Press, Oxford (1980), pp. 149-151.

6. H. Jeffreys and B. S. Jeffreys, Methods of Mathematical Physics, 3rd ed., Cambridge University Press, Cambridge (1956), pp. 498-500.

7. P. G. Shewmon, Diffusion in Solids, McGraw-Hill, New York (1963), pp. 31-32.